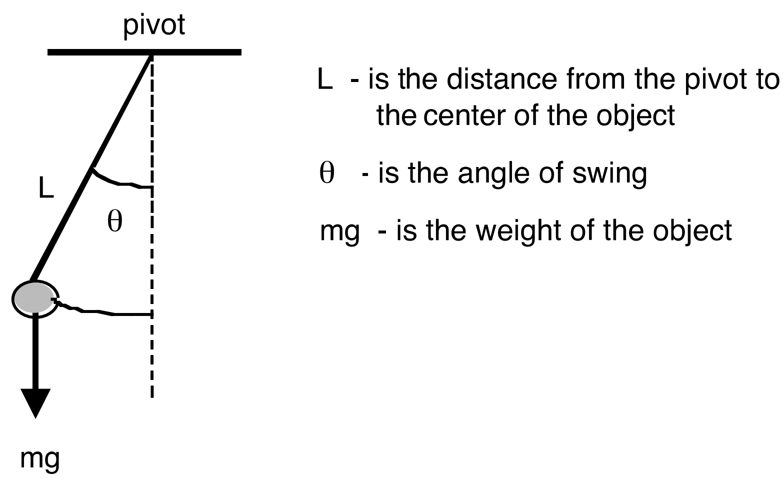
Lab 9: Pendulum (M9)

Objectives

- Examine a simple pendulum with changing length and mass.
- Understand how damping changes the amplitude of oscillations.
- Examine a physical pendulum.

Theory

A **simple pendulum** consists of a particle of mass m, attached to a frictionless pivot by a cable of length L. The mass of the cable should be negligible. When the particle is pulled away from its equilibrium position by an angle θ and released, the particle swings back and forth. By attaching this device to a computer, we can record the position of the particle as time passes. The graphical pattern produced by the computer is similar (but not identical) to the sinusoidal pattern for a simple harmonic motion. The period of oscillations recorded by the computer from the graph is given by $T = \frac{\Delta t}{\#\text{ of oscillations}}$.



If the simple pendulum amplitude is a **small angle**, then its motion resembles that for a linear, **harmonic oscillator** (e.g., a block on a spring). Therefore, we would expect the period of oscillations of the back-and-forth movement of the pendulum to be given by an equation analogous to $T = 2\pi\sqrt{\frac{m}{k}}$, where k is the spring constant of a block-spring oscillator. Instead of the spring constant k, another constant also called $k = mgh$ will appear, and (as usual in rotational motion) moment of inertia I replaces the mass m.

$$T = 2\pi\sqrt{\frac{I}{mgh}} \qquad \text{(for small angles)} \qquad (1)$$

The moment of inertia of a small particle of mass *m*, rotating at a radius $r = L$ about an axis, is given by $I = mL^2$. Substituting this expression for *I* into equation (1) and substituting $h = L$ reveals that the period of oscillations for a simple pendulum is given by the following equation valid for small angles.

$$I = mL^2 \text{ and } h = L \;\Rightarrow\; T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{mL^2}{mgL}} = 2\pi\sqrt{\frac{L}{g}} \qquad \text{(for small angles)} \qquad (2)$$

In a simple harmonic motion, an object has constant amplitude of oscillations, because there is no mechanism for dissipating the energy. In reality, friction or some other energy-dissipating mechanism is always present. In the presence of energy dissipation, the amplitude of oscillations decreases as time passes, and the motion is no longer a simple harmonic motion. Instead, it is referred to as **damped harmonic motion**, the decrease in amplitude being called "*damping*".

It is not necessary that an object swinging back and forth be a small particle. It may be an extended object; in which case the pendulum is called a **physical pendulum**. For small oscillations, equation (1) still applies, but the moment of inertia *I* is no longer mL^2, but it depends on the shape and size of the oscillating object. For a rod suspended on its end, the moment of inertia $I = \frac{1}{3}mL^2$. In addition, the height *h* of a physical pendulum is equal to the **distance between the axis at the pivot and the center of gravity of the object**, for the rod in use it is $h = \frac{L}{2}$.

Therefore, the period of oscillations for a rod suspended on its end is given by the following equation:

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{\frac{1}{3}mL^2}{mg\frac{L}{2}}} = 2\pi\sqrt{\frac{2L}{3g}} \qquad \text{(for small angles)} \qquad (3)$$

This formula looks similar to the simple pendulum formula, except of the $\sqrt{2/3}$ factor.

Angular position or displacement is usually measured in radians (rad). The conversion of angle from radians to degree is given by the following.

$$\theta = 2\pi\ (\text{rad}) = 360° \quad \text{and} \quad 1 \text{ rad} = \frac{360°}{2\pi} \cong 57.3° \;, \quad 1° = 0.01745 \text{ rad}$$

Procedure:

Login using your Purdue career account. Download files for experiment M9 from Brightspace for Physics 220. Double-click on the "M9 Activity 1" icon.

Activity 1: Simple Pendulum

A. Use the **black, short rod** and attach the **brass mass** to the end of the rod. You should attach the brass mass so that the bottom of the brass mass is flush with the end of the rod. Measure the length (L) of the rod <u>from the center of the pulley (the pivot point) to the center of the smaller brass mass</u> and record it on your data sheet.

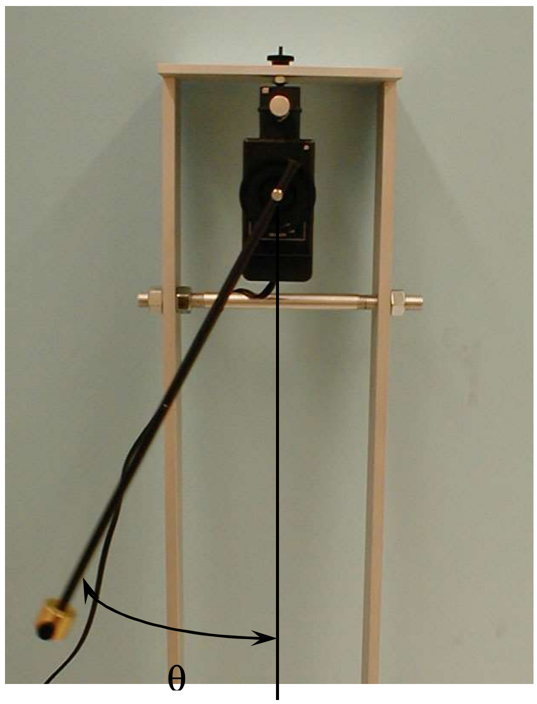


Figure 1.

Using the computer, record one set of data. **Start recording data with stationary pendulum** in its equilibrium (vertical) position. Displace the pendulum by a small angle ($\theta \leq 10° = 0.18$ rad), but big enough so the oscillations will not die off too quickly. <u>Promptly</u> release the pendulum and let is swing freely. PASCO software assumes the <u>first recorded angular position</u> to be "0 radians". Therefore, if you start with pendulum in its equilibrium (vertical position), the angular position vs. time graph should be nicely centered on the equilibrium position "0 radians".

Use the "Smart Tool" icon to read the time of the **two points separated by ten periods** (see Figure 2). Record these numbers on your data sheets. Calculate the average period of oscillations T_{AV}. Since we measured the time of ten consecutive periods and then divide it by ten, it is like making ten individual period measurements and then calculating the average value.

Print a copy of the oscillations on the screen by clicking once in the window to make it active and then selecting **Print** from the **File** menu. Label this graph "*Simple Pendulum*" and write your name on the graph. Mark the selected maximum (or minimum) points with a pen (for example, circle them).

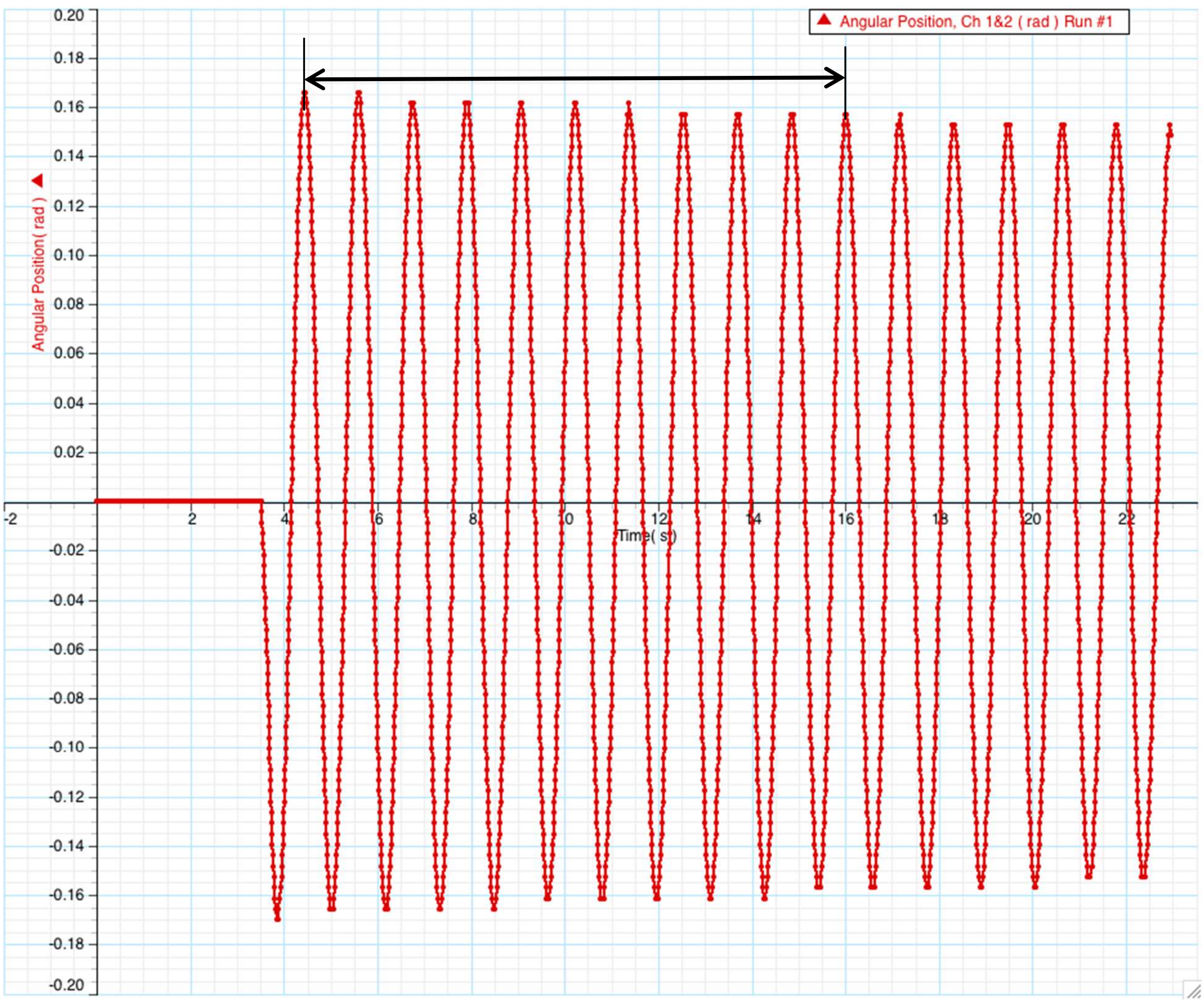


Figure 2.

B. In this part, we will use the average period of oscillations measured in Part A to calculate the acceleration due to gravity g_{exp} and to estimate the mass of the Earth - M_{Earth}.

First, use equation (2) to derive the formula for the acceleration due to gravity, when we know the period of oscillations T and the length of the pendulum L. Write the formula on the data sheets.

Next, use the length of your pendulum L from Part A and the average period of oscillations T_{AV} to calculate the acceleration due to gravity. Calculate the percent difference Δg between the measured value of the acceleration g_{exp} and the value given in the textbook ($g = 9.80$ m/s^2).

Next, use the measured acceleration g_{exp} to estimate the mass of the Earth: M_{Earth}.

The acceleration due to gravity can also be written as:

$$g_{\exp} = G\frac{M_{Earth}}{r^2} \tag{4}$$

Using Eq. (4), we can derive the expression for the mass of the Earth - M_{Earth}

$$g_{\exp} = G\frac{M_{Earth}}{r^2} \quad \Rightarrow \quad M_{Earth} = \frac{g_{\exp} * r^2}{G}$$

The mean radius of the Earth is approximately equal to $r = 6.36*10^6$ m and the universal gravitational constant G is equal to $G = 6.67*10^{-11}$ Nm^2/kg^2. Use the measured acceleration g_{exp} to estimate the mass of the Earth: M_{Earth}.

Activity 2: Damped Pendulum

Select **Open Activity** from the **File**. **Do not save any changes.** Next, select: "M9 Activity 2" and open it. An angular position graph should appear.

Leave the brass mass on the end of the rod. Take the rubber O-ring and run it up the silver rod until it reaches the pulley. Wrap the rubber O-ring around the pulley and the metal rod as shown in Figure 3.

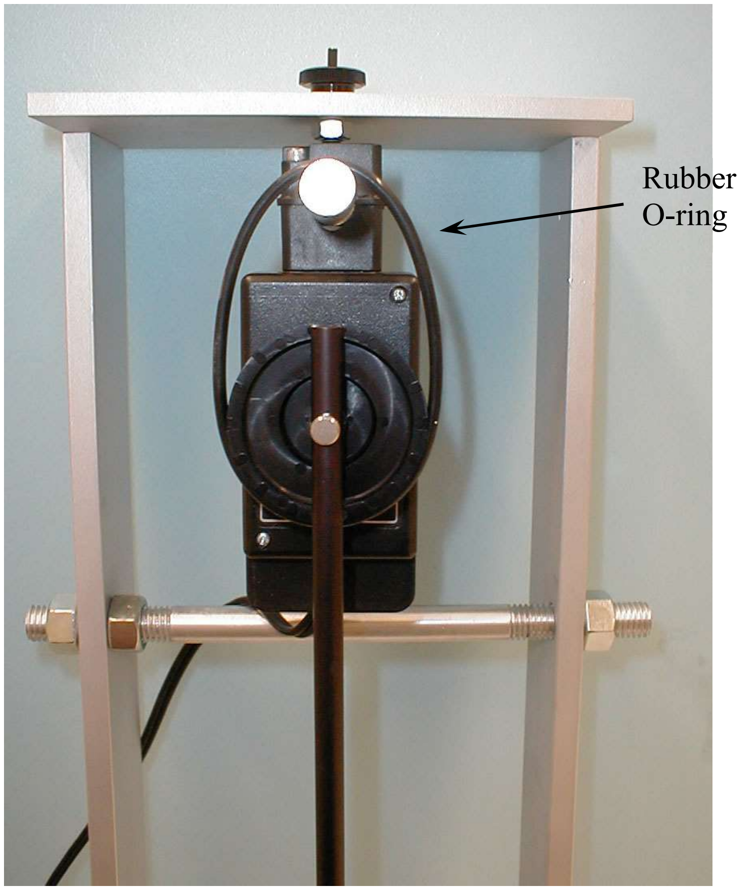
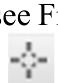


Figure 3.

In this activity, you will <u>not</u> measure the period of oscillations, but the amplitude of the oscillations as a function of time. Therefore, you should **use a much larger initial displacement angle (θ ≅ 30° or ~0.5 rad)**. For a small initial angle, the damping effect would cause the pendulum to slow down too rapidly. Because of the high displacement and the length of the rod, be careful when it swings back and forth.

Start recording data with stationary pendulum in its equilibrium (vertical) position. Displace the pendulum approximately θ ≅ 30° (or ~0.5 rad) of the equilibrium and take one good data run, see Figure 4. Record the first five maximums of angular displacement from the graph. Use the  icon to read the values from the graph. The maximum of oscillations is called the **amplitude** of oscillations. **Print** a copy of the graph and label it "*Damped Pendulum*". Write your name on the graph and record the first five amplitudes of angular displacement.

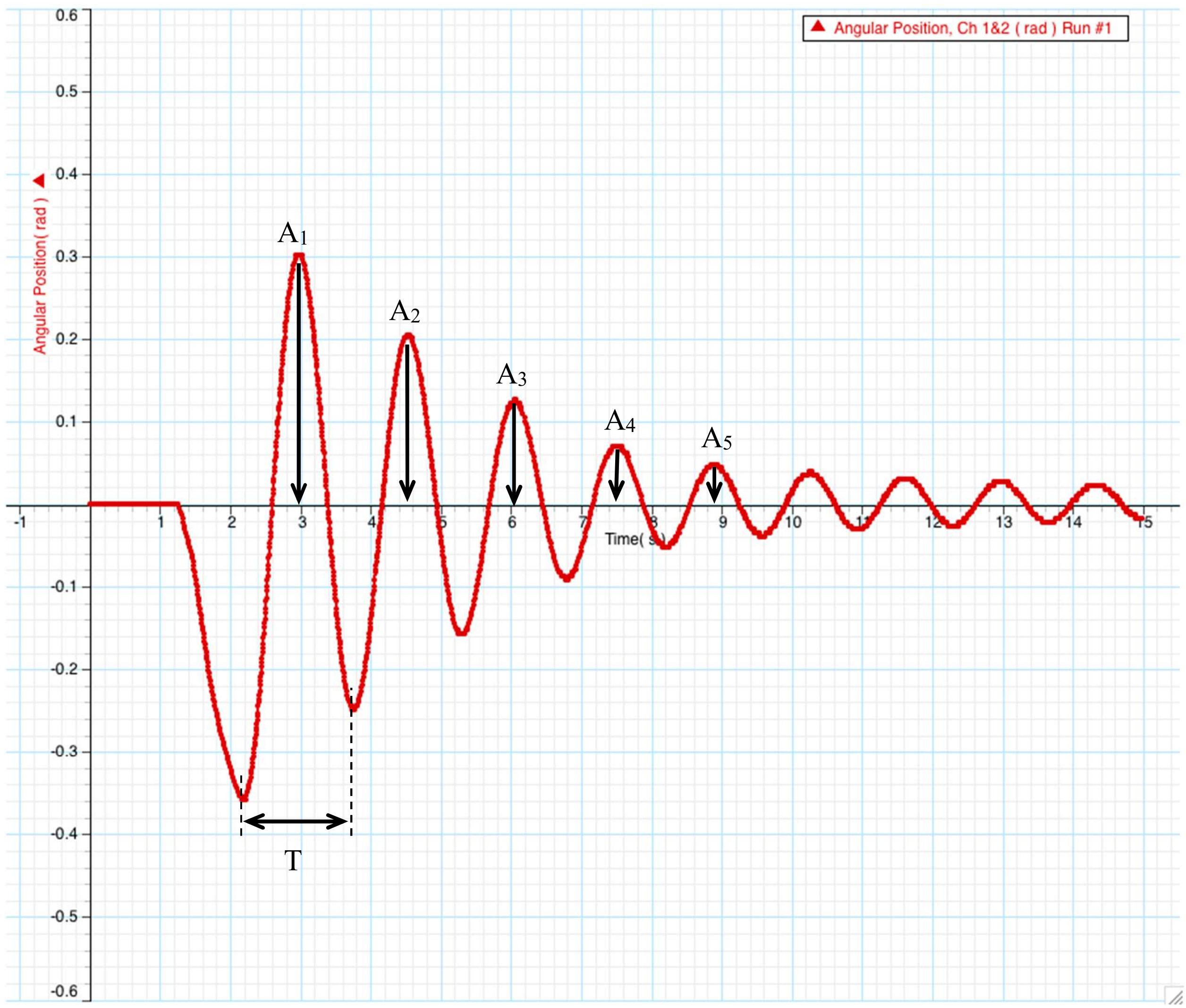


Figure 4.

For a simple pendulum, the amplitude of oscillations does not change with time. For a damped pendulum with some sort of friction involved, the amplitude of oscillations decreases with time. That decrease of amplitude is described by the exponential function:

$$A(t) = A_0 \exp(-bt),$$

where A is the amplitude, t is time, and b is a damping coefficient that is proportional to the friction force. The stronger damping the larger damping coefficient b. If T is the period of oscillations, then the consecutive amplitudes A_1, A_2, A_3, *etc.,* are given by the following.

$A_1 = A(t_0 + T)$, $A_2 = A(t_0 + 2T)$, $A_3 = A(t_0 + 3T)$, etc.

The ratio of two consecutive amplitudes should be a constant value for all amplitudes!

$$\frac{A_{n+1}}{A_n} = \frac{A\left(t_0 + (n+1)T\right)}{A\left(t_0 + nT\right)} = \frac{A_0 \exp\left(-b\left(t_0 + nT + T\right)\right)}{A_0 \exp\left(-b\left(t_0 + nT\right)\right)} = \frac{\exp\left(-b\left(t_0 + nT\right)\right)\exp\left(-bT\right)}{\exp\left(-b\left(t_0 + nT\right)\right)}$$

$$\frac{A_{n+1}}{A_n} = \exp\left(-bT\right) = const.$$

The **energy** of oscillations E is proportional to the amplitude squared, i.e., $E \sim A^2$.

$$\frac{E_{n+1}}{E_n} = \left(\frac{A_{n+1}}{A_n}\right)^2$$

Therefore, the E_2/E_1, E_3/E_2, E_4/E_3, E_5/E_4 ratios should all have **the same value**. In real world, that theoretical conclusion is fulfilled only approximately because it is exact only for small angles (i.e., for small amplitudes).

Calculate the E_2/E_1, E_3/E_2, E_4/E_3, E_5/E_4 ratios and find the average value of the E_{n+1}/E_n ratio.

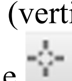
What percentage of the energy is lost during each period of oscillations?

Activity 3: Physical Pendulum

Select **Open Activity** from the **File** menu. **Do not save any changes.** Next, select: "M9 Activity 3" and open it.

Remove the rubber O-ring and the brass mass used in *Activity 2*. Replace the black rod with the **blue rod**. **Do not attach any additional masses**. Measure the length (L) of the rod from the center of the pulley to the end of the rod.

In this activity, you will use the blue rod without any masses attached). For all previous activities, the mass of the brass attachment was much larger than the mass of the rod. Therefore, we could use the simple pendulum model. Here the mass of the pendulum is equally distributed along the rod. The simple pendulum model does not describe this type of pendulum accurately. Therefore, we should use equations given by the **physical pendulum model** (see the "Theory" section).

We are going to measure the period of oscillations in the same way as we did for the simple pendulum in Activity 1. Using the computer, record one set of data. **Start recording data with stationary pendulum** in its equilibrium (vertical) position. Displace the pendulum by a small angle ($\theta \leq 10° = 0.175$ rad). Use the  icon to read the time of the two points on the graph separated by ten periods. Record these numbers on your data sheets. Calculate the average period of oscillations.

Using the equation for a physical pendulum (3), calculate the theoretical value of the period of oscillations. Calculate the absolute and percent differences between the experimental (average) and the theoretical values for the period of oscillations. Note that we do not really need the mass of the rod to calculate the period of oscillations using equation (3).

Compare the predictions of the simple pendulum model with the predictions of the physical pendulum model and with your experimental data. Which theoretical model works better for the pendulum made with a solid rod without any additional masses attached?

Remove the blue rod from the pendulum and re-attach the short, black rod used in Activity 1.

Complete the lab report and return it to the lab TA.

Make sure to complete the following tasks:

You must submit the answers to the prelaboratory questions online. (3.5 points)

1. Your completed Data Sheets. (4.5 points)
2. The two graphs printed out during *Activities* 1 and 2. ($2 \times$ 1p.=2 points)
3. Return the completed lab report to your lab TA.